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Temperature Distributions in Semitransparent Coatings—A Special Two-Flux Solution

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Radiative transfer is analyzed in a semitransparent coating on an opaque substrate and in a semitransparent layer for evaluating thermal protection behavior and ceramic component performance in high-temperature applications. Some ceramics are partially transparent for radiative transfer, and at high temperatures internal emission and reflections affect their thermal performance. The behavior is examined for a ceramic component for which interior cooling is not provided. Two conditions are considered: 1) the layer is heated by penetration of radiation from hot surroundings while its external surface is simultaneously film cooled by convection, and 2) the surface is heated by convection while the semitransparent material cools from within by radiant emission leaving through the surface. By using the two-flux method, which has been found to yield good accuracy in previous studies, a special solution is obtained for these conditions. The analytical results include isotropic scattering and require only an integration to obtain the temperature distribution within the semitransparent material. Illustrative results are given to demonstrate the nature of the thermal behavior.

Nomenclature

a = absorption coefficient of semitransparent material, m^{-1}
C = integration constant in energy equation, W/m; $\tilde{C} = C/D\sigma T_g^4$
D = thickness of coating or one-half of layer (Fig. 1), m
G = flux quantity defined in Eq. (2b), W/m^2
$\tilde{G}$ = dimensionless flux quantity, $G/\sigma T_g^4$
H = dimensionless convection–radiation parameter, $h/\sigma T_g^3$
h = heat transfer coefficient at boundary, W/m^{2}K
K = extinction coefficient, $a + \sigma_s$, m^{-1}
k = thermal conductivity, W/m·K
N = conduction-radiation parameter, $k/\sigma T_g^3 D$
n = refractive index of layer
q = heat flux, W/m^2
q_r = radiative heat flux, W/m^2; $\tilde{q}_r = q_r/\sigma T_g^4$
q_r^o = externally incident radiation flux, W/m^2; $\tilde{q}_r^o = q_r^o/\sigma T_g^4$
q_r^+, q_r^- = radiative fluxes in + and − x directions, W/m^2
T = absolute temperature, K
T_g = gas temperature at coating surface, K
t = dimensionless temperature, T/T_g
X = dimensionless coordinate, x/D
x = coordinate in semitransparent layer (Fig. 1), m
ε_D = emissivity of substrate
κ_D = optical thickness, KD
ρ = diffuse reflectivity of interface
σ = Stefan-Boltzmann constant, W/m^{2}K^4
σ_s = scattering coefficient in layer, m^{-1}
Ω = scattering albedo in layer, $\sigma_s/(a + \sigma_s) = \sigma_s/K$

Subscript
tot = total heat flux by combined conduction and radiation

Superscripts
i = inside of layer
o = outside of layer

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Introduction

THE development of ceramic coatings to protect materials for high-temperature use is critical for advanced aircraft engines where high thermal efficiency is required. Some coatings partially transmit radiant energy in certain wavelength regions. In high-temperature surroundings such as in a combustion chamber, infrared and visible radiation transmitted within the coating provides internal heating that affects temperatures of the coating and its substrate.

At elevated temperatures, radiant emission within a material can be large. For a material with a high refractive index, this is especially true since internal emission depends on the refractive index squared. Since radiation leaving through an interface cannot exceed that from a blackbody, internal reflections occur when radiation passes into a material with a lower refractive index. In addition to internal emission, absorption, and reflection, radiant scattering and heat conduction take part in the energy transfer. The interaction of internal radiation and conduction must be understood for semitransparent components and protective coatings subjected to radiative and convective environments. Internal scattering must be examined as it can influence the temperature distribution for some conditions.

An important area involving radiation within hot materials with refractive indices larger than one and with significant internal emission is the heat treating and cooling of glass plates.[1] The related literature has been briefly reviewed in previous papers.[2,3] In these papers, temperature distributions and heat flows in partially transmitting materials with high refractive indices are predicted using the radiative transfer equations coupled with heat conduction. The resulting integral equations, including the scattering source function for some of the work, are solved numerically. Each exterior boundary is heated by radiation and convection, and diffuse interface reflections are included. For comparisons during the development of approximate solutions in Ref. 4, the numerical solutions were extended to a three-layer composite with three spectral bands in each layer and with isotropic scattering. This simulates a ceramic layer with a reinforcing layer, or with coatings of other ceramic materials for protection from corrosive atmospheres such as combustion gases.

Since the formulation and solution of the exact radiative transfer equations including scattering is rather complex, it is desirable to develop more convenient approximate methods such as the two-flux method if these can provide accurate results. The two-flux equations are given in Refs. 5 and 6. The two-flux method was shown to give accurate results for a gray layer with a refractive index of one between boundaries with specified temperatures.[7,8] Two-flux and diffusion solutions, and combinations of the two for layers with optically thin and thick spectral bands, were derived in Ref. 4 for materials with refractive indices larger than one. This included heating conditions where the boundary temperatures are not specified and must be determined during the solution. The two-flux formulation yielded very accurate results in comparisons with exact solutions. In Ref. 9 two-flux solutions were obtained with two spectral bands for a packed bed with two layers of particles.

In the present work it is shown that for some types of heating and cooling conditions, the two-flux solution is reduced to a simple integration to generate the material temperature distribution. Results are given that illustrate the behavior of a semitransparent protective coating on an opaque substrate when the substrate is not internally cooled. This models a protected component that is either heated on all sides by hot gas while it is cooled by radiating to cold surroundings, or is being film cooled on all sides while being subjected to radiation from hot surroundings.

Analysis

Energy and Two-Flux Equations

Figure 1 shows the geometries and conditions for which the present analysis was developed. In Fig. 1a a semitransparent layer such as a ceramic component is in a high-temperature environment that provides symmetric radiative and convective conditions at both boundaries. In Fig. 1b there is a semitransparent protective coating of thickness D on each side of a high-temperature opaque component. Both sides of the component have the same coating and are subjected to the same external conditions; from this symmetry there is no net heat flow through the entire coated composite. The results also apply for a coating on one side of an opaque substrate where the uncoated side is insulated or has negligible heat loss. The layer in Fig. 1a and the coatings in Fig. 1b are semitransparent and are gray emitting, absorbing, scattering, and heat-conducting materials with refractive indices larger than one. The convective and radiative conditions are uniform on the coating boundaries. The temperature distribution is to be found in the semitransparent layer of Fig. 1a and in the coating material in Fig. 1b, and how the distribution depends on the parameters will be illustrated.

From symmetry, the energy equation combining the energy fluxes by conduction and radiation within the layers in Fig. 1 is

$$-k\left.\frac{dT}{dx}\right|_x + q_r(x) = q_{tot} = 0 \tag{1}$$

Thermal properties are assumed independent of temperature. Since the substrate in Fig. 1b is opaque it does not have an internal radiative energy source. Then from Eq. (1) with $q_r(x) = 0$, $dT/dx = 0$ throughout the substrate, yielding a uniform substrate temperature equal to $T(D)$. This is true for any substrate thermal conductivity.

The solution method developed here to determine the heat transfer behavior of the semitransparent material is based on

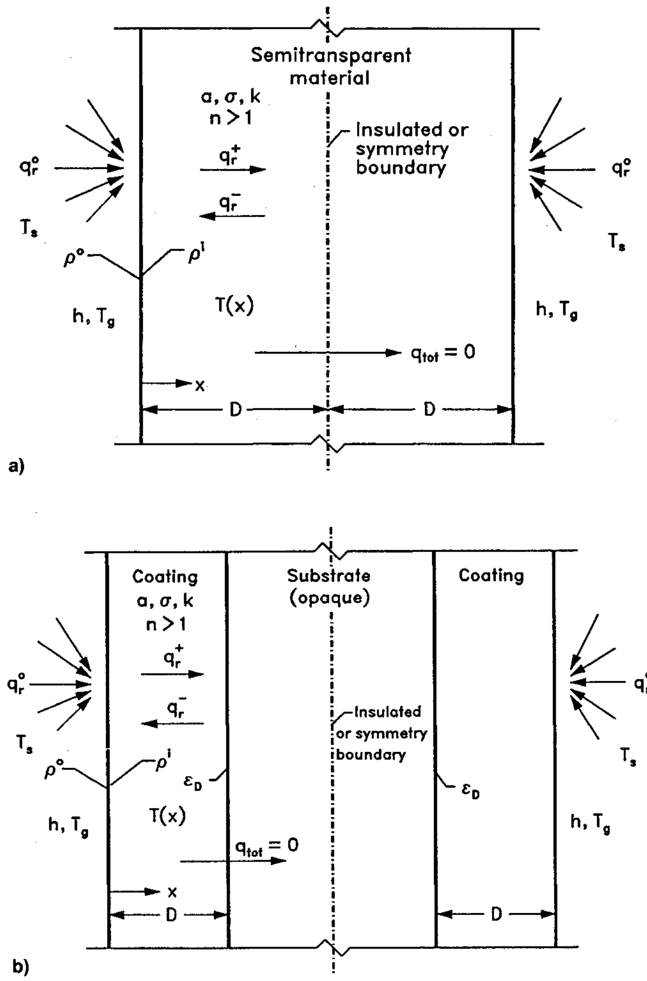

Fig. 1 Geometry and coordinate system for two-flux analysis with symmetric thermal boundary conditions: a) single semitransparent layer and b) symmetric semitransparent coatings on an opaque substrate.

the two-flux method[4,5] for the radiative fluxes. To validate this approximate method some results are compared with numerical solutions of the exact radiative transfer equations using the computer program from Ref. 3. In the two-flux method, the radiative fluxes $q_r^+(x)$ and $q_r^-(x)$ are in the positive and negative directions (Fig. 1), and each flux is assumed isotropic. The radiative flux $q_r(x)$ in the x direction and a quantity $G(x)$ are related to $q_r^+(x)$ and $q_r^-(x)$ by[5]

$$q_r(x) = q_r^+(x) - q_r^-(x) \tag{2a}$$

$$G(x) = 2[q_r^+(x) + q_r^-(x)] \tag{2b}$$

Solving Eqs. (2a) and (2b) for $q_r^+(x)$ and $q_r^-(x)$ gives the useful relations

$$q_r^+(x) = \tfrac{1}{2}\{[G(x)/2] + q_r(x)\} \tag{3a}$$

$$q_r^-(x) = \tfrac{1}{2}\{[G(x)/2] - q_r(x)\} \tag{3b}$$

The two-flux equations including isotropic scattering are[5]

$$\frac{dq_r(x)}{dx} = K(1-\Omega)[4n^2\sigma T^4(x) - G(x)] \tag{4}$$

$$\frac{dG(x)}{dx} = -3Kq_r(x) \tag{5}$$

Integration of Energy and Two-Flux Equations

The $q_r(x)$ in Eq. (5) is substituted into Eq. (1), which is then integrated to give

$$kT(x) + [1/(3K)]G(x) = C \tag{6}$$

The two-flux relation Eq. (4) is now used. The energy equation Eq. (1) is differentiated and used to eliminate $dq_r(x)/dx$ on the left side of Eq. (4). The $G(x)$ on the right side of Eq. (4) is eliminated by using Eq. (6). This yields the following equation in terms of $T(x)$:

$$k\frac{d^2T}{dx^2} = K(1 - \Omega)[4n^2\sigma T^4(x) + 3KkT(x) - 3KC] \tag{7}$$

The order of Eq. (7) is reduced by multiplying the entire equation by dT/dx and integrating each term. The constant of integration is eliminated by subtracting the integrated equation evaluated at $x = 0$ to give

$$\frac{k}{2}\left[\left(\frac{dT}{dx}\right)_x^2 - \left(\frac{dT}{dx}\right)_{x=0}^2\right] = K(1 - \Omega)\left\{\frac{4}{5}n^2\sigma[T^5(x) - T^5(0)] + \frac{3}{2}Kk[T^2(x) - T^2(0)] - 3KC[T(x) - T(0)]\right\} \tag{8}$$

Boundary Conditions

At the boundary $x = 0$, external convection is equal to conduction within the layer; radiation does not enter this balance since radiation is a volume process and, hence, for a semitransparent material, there is no absorption or emission at the surface itself. Then, at $x = 0$

$$-k\left.\frac{dT}{dx}\right|_{x=0} = h[T_g - T(0)] \tag{9}$$

To derive the interface condition at $x = D$ in Fig. 1b, the radiative flux leaving the opaque gray substrate at $x = D$ is expressed in terms of the radiation emitted and reflected into the coating as

$$q_r^-(D) = \varepsilon_D n^2\sigma T^4(D) + (1 - \varepsilon_D)q_r^+(D) \tag{10}$$

Some substitutions into Eq. (10) are now made. The $q_r^+(D)$ and $q_r^-(D)$ are eliminated by using Eqs. (3a) and (3b). The $G(D)$ and $q_r(D)$ that this introduces are eliminated by using Eqs. (6) and (1). The result is a relation for $dT/dx|_{x=D}$ in terms of $T(D)$ and the constant C

$$k\left.\frac{dT}{dx}\right|_{x=D} = \frac{2\varepsilon_D}{2 - \varepsilon_D}\left[\frac{3}{4}CK - \frac{3}{4}kKT(D) - n^2\sigma T^4(D)\right] \tag{11}$$

For the symmetric single layer in Fig. 1a, $dT/dx|_{x=D} = 0$, which is also equivalent to the limit in Eq. (11) when $\varepsilon_D \to 0$.

To obtain an expression for C, Eq. (6) is used at $x = 0$ to give $C = kT(0) + [1/(3K)]G(0)$. The $G(0)$ depends on the externally incident radiation and on the flux internally reflected at the boundary; an expression for $G(0)$ is derived in Ref. 4 as

$$G(0) = 4\frac{1 - \rho^o}{1 - \rho^i}q_r^o - 2\frac{1 + \rho^i}{1 - \rho^i}q_r(0) \tag{12}$$

Using Eq. (12), C is given by

$$C = kT(0) + \frac{1}{3K}\left[4\frac{1 - \rho^o}{1 - \rho^i}q_r^o - 2\frac{1 + \rho^i}{1 - \rho^i}q_r(0)\right] \tag{13}$$

The $q_r(0)$ is eliminated from Eq. (13) by using Eq. (1) at $x = 0$ and replacing the conduction term with the convection term in Eq. (9); this yields

$$C = kT(0) + \frac{1}{3K}\left\{4\frac{1 - \rho^o}{1 - \rho^i}q_r^o + 2\frac{1 + \rho^i}{1 - \rho^i}h[T_g - T(0)]\right\} \tag{14}$$

Form of Differential Equation for Integration

The boundary condition, Eq. (9), is substituted into Eq. (8) and the result is solved for dT/dx to yield

$$\frac{dT}{dx} = \pm\left[\frac{2K}{k}(1 - \Omega)\left\{\frac{4}{5}n^2\sigma[T^5(x) - T^5(0)] + \frac{3}{2}Kk[T^2(x) - T^2(0)] - 3KC[T(x) - T(0)]\right\} + \left\{\frac{h}{k}[T_g - T(0)]\right\}^2\right]^{1/2} \tag{15}$$

The sign in front of the square root is selected according to the initial slope of the temperature profile expected from the heating and cooling imposed by external radiation and convection.

Before discussing the numerical procedure, the differential equation, constant C, and conditions at $x = D$ for Figs. 1a and 1b are placed in dimensionless form:

$$\frac{dt}{dX} = \pm\left[\frac{2\kappa_D}{N}(1 - \Omega)\left\{\frac{4}{5}n^2[t^5(X) - t^5(0)] + \frac{3}{2}\kappa_D N[t^2(X) - t^2(0)] - 3\kappa_D\tilde{C}[t(X) - t(0)]\right\} + \left\{\frac{H}{N}[1 - t(0)]\right\}^2\right]^{1/2} \tag{16a}$$

$$\tilde{C} = Nt(0) + \frac{1}{3\kappa_D}\left\{4\frac{1 - \rho^o}{1 - \rho^i}\tilde{q}_r^o + 2\frac{1 + \rho^i}{1 - \rho^i}H[1 - t(0)]\right\} \tag{16b}$$

$$\left.\frac{dt}{dX}\right|_{X=1} = 0 \tag{17a}$$

$$\left.\frac{dt}{dX}\right|_{X=1} = \frac{1}{N}\frac{2\varepsilon_D}{2 - \varepsilon_D}\left[\frac{3}{4}\tilde{C}\kappa_D - \frac{3}{4}N\kappa_D t(1) - n^2t^4(1)\right] \tag{17b}$$

Numerical Solution

To obtain the temperature distribution, the $\tilde{C}$ in Eq. (16b) is substituted into Eq. (16a); dt/dX is then a function of $t(0)$ and the specified parameters. A value of $t(0)$ is guessed and forward integration is performed from $X = 0$ to 1 using a fourth-order Runge–Kutta method. At the end of the integration the condition in Eq. (17a) or (17b) is checked to see if it is satisfied; if not, $t(0)$ is adjusted until it is satisfied. For some values of the parameters the integration is very sensitive to $t(0)$; double precision was used for these cases. If the optical thickness is large there is radiative penetration for only a small distance and the temperature distribution is uniform except in a small region near the surface where radiation enters. Then the forward integration is performed only over the X distance where the temperature is changing significantly, and the particular $t(0)$ is found for which $dt/dX \to 0$ as the integration proceeds. For these conditions, the temperature distribution is independent of the interface condition at $X = 1$ in Eq. (17b), and $dt/dX|_{X=1} = 0$, which also corresponds to the symmetric single layer in Fig. 1a.

Within the substrate in Fig. 1b, the temperature is uniform and equals $t(1)$ at the coating-substrate interface. This follows from the energy equation in the absence of heat sources within the opaque substrate, and for the symmetric conditions considered here.

Special Case for a Coating at Uniform Temperature

In some applications a protective coating has a high thermal conductivity or is thin enough that its temperature does not vary much throughout its thickness. The following analysis is for the special case of a layer at uniform temperature.

Equation (4) is differentiated with T constant and is substituted into the left side of Eq. (5) to eliminate $dG(x)/dx$

$$\frac{d^2q_r(x)}{dx^2} = 3K^2(1 - \Omega)q_r(x) \tag{18}$$

The general solution of Eq. (18) is

$$q_r(x) = C_1e^{\alpha x} + C_2e^{-\alpha x} \qquad \alpha = K[3(1 - \Omega)]^{1/2} \tag{19}$$

Substituting $q_r(x)$ into Eq. (4) gives for $G(x)$

$$G(x) = 4n^2\sigma T^4 - [3/(1 - \Omega)]^{1/2}(C_1e^{\alpha x} - C_2e^{-\alpha x}) \tag{20}$$

An overall energy balance states that incident radiation and convection to the coating must equal reflected radiation and the radiation leaving the coating: $q_r^o + h[T_g - T(0)] = q_r^o\rho^o + q_r^-(0)(1 - \rho^i)$. By applying the radiative conditions across $x = 0$, $q_r(0) = q_r^+(0) - q_r^-(0) = q_r^o(1 - \rho^o) + \rho^i q_r^-(0) - q_r^-(0)$, the energy balance reduces to

$$T(0) = [q_r(0)/h] + T_g \tag{21}$$

Thus, the uniform coating temperature $T = T(0)$ can be obtained by finding $q_r(0)$ inside the coating from Eq. (19) by determining C_1 and C_2 from the boundary conditions applied to Eqs. (19) and (20). At $x = 0$ and D, Eqs. (12) and (10) apply. Using Eqs. (3a) and (3b), Eq. (10) becomes

$$G(D) = 2[(2 - \varepsilon_D)/\varepsilon_D]q_r(D) + 4n^2\sigma T^4 \tag{22}$$

Equations (19) and (20) are substituted into Eqs. (12) and (22) to give two simultaneous equations that are solved for C_1 and C_2. Then $q_r(0) = C_1 + C_2$ from Eq. (19) is inserted into Eq. (21); the result is placed into dimensionless form to obtain the following equation that can be evaluated for t with a root solver:

$$t = 1 + \frac{1}{H} \times \left\{\frac{[(\beta_1 - \beta_3)e^{-2\beta_2\kappa_D} + \beta_1 + \beta_3]4(\beta_5\bar{q}_r^o - n^2t^4)}{(-\beta_1 + 2\beta_4)(\beta_1 - \beta_3)e^{-2\beta_2\kappa_D} + (\beta_1 + 2\beta_4)(\beta_1 + \beta_3)}\right\} \tag{23}$$

where

$$\beta_1 = \left(\frac{3}{1 - \Omega}\right)^{1/2}, \qquad \beta_2 = [3(1 - \Omega)]^{1/2}$$

$$\beta_3 = \frac{4 - 2\varepsilon_D}{\varepsilon_D}, \qquad \beta_4 = \frac{1 + \rho^i}{1 - \rho^i}, \qquad \beta_5 = \frac{1 - \rho^o}{1 - \rho^i}$$

Limit for a Transparent Coating on an Opaque Substrate

For $\kappa_D \to 0$ radiation is absorbed and emitted only by the substrate, but this absorption and emission is affected by multiple reflections within the coating. The net energy absorbed by the substrate at $X = 1$ is conducted out through the coating and is equal to the energy convected from the coating surface. The coating temperature distribution is linear, and $t(1)$ and $t(0)$ are given by

$$\varepsilon_D\left[\frac{\bar{q}_r^o(1 - \rho^o) + \varepsilon_D\rho^i n^2t^4(1)}{1 - \rho^i(1 - \varepsilon_D)} - n^2t^4(1)\right] = H[t(1) - 1]\left(\frac{H}{N} + 1\right)^{-1} \tag{24a}$$

$$t(0) = \left[t(1) + \frac{H}{N}\right]\left(\frac{H}{N} + 1\right)^{-1} \tag{24b}$$

Apparent Surface Temperature from Emitted Radiation

For temperature measurements using pyrometry, it is of interest to examine the coating surface temperature calculated from the radiation leaving the coating as compared with the actual $t(0)$ obtained from the solution of Eq. (16a). For a gray material the transmittance or absorptance of the coating surface is $1 - \rho^o$. From an overall energy balance the energy leaving the coating must equal the incident radiation transmitted across the surface and the energy transferred to the surface by convection. Then the apparent temperature T_{app} required to emit this energy, relative to the actual surface temperature $T(0)$, is given by

$$\frac{T_{app}}{T(0)} = \frac{1}{t(0)}\frac{1}{(1 - \rho^o)^{1/4}}\{\bar{q}_r^o(1 - \rho^o) + H[1 - t(0)]\}^{1/4} \tag{25}$$

Results and Discussion

To illustrate the solution behavior, temperature distributions are given for a symmetric semitransparent layer as in Fig. 1a, and for a semitransparent protective coating on an insulated or symmetric opaque substrate as in Fig. 1b. Results are given for two situations: radiative heating with convective cooling (a film-cooled material in hot surroundings), and convective heating with radiative cooling (a material in a hot gas stream with cooled surrounding surfaces). The results illustrate how the temperature of a substrate, which is to be protected thermally or from a corrosive atmosphere, is influenced by the coating physical parameters.

Symmetric Heating Conditions on a Semitransparent Layer

Results are given first to validate the two-flux method by comparison with numerical solutions of the radiative transfer equations. By using the computer program from Ref. 3 with symmetric boundary conditions, temperature distributions are obtained for the layer in Fig. 1a, or for the coating in Fig. 1b in the limit of either a perfectly reflecting substrate or when $t(X)$ is independent of substrate emissivity as will be discussed. For these conditions $dt/dX|_{X=1} = 0$. In Fig. 2 the temperature distributions are for the semitransparent material exposed to a hot environment that provides an incident radiative flux $q_r^o = \sigma(1.5T_g)^4$. The surface is being rather strongly convectively cooled with a convection-radiation parameter $H = 10$. Radiation penetrates into the layer and the net absorbed energy is conducted back to the surface at $X = 0$ and removed by convection.

The results from the numerical solution of the exact transfer equations show that the two-flux analysis provides excellent agreement, even in regions with large temperature gradients. The two-flux solution will then be used to illustrate some aspects of the thermal performance of a coating on an opaque substrate for the boundary conditions considered here. The results in Fig. 2 are valid for a coating as in Fig. 1b when $t(X)$

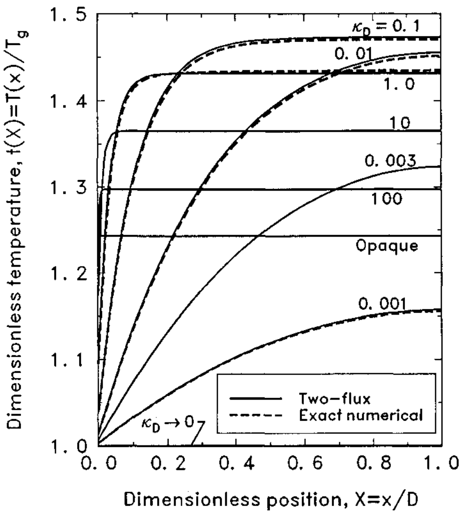

Fig. 2 Effect of optical thickness on temperature distributions in a semitransparent layer symmetrically heated by radiation and cooled by convection (n = 1.5, t_g = 1, $\bar{q}_r^o$ = 1.5^4, H = 10, N = 0.1, and Ω = 0).

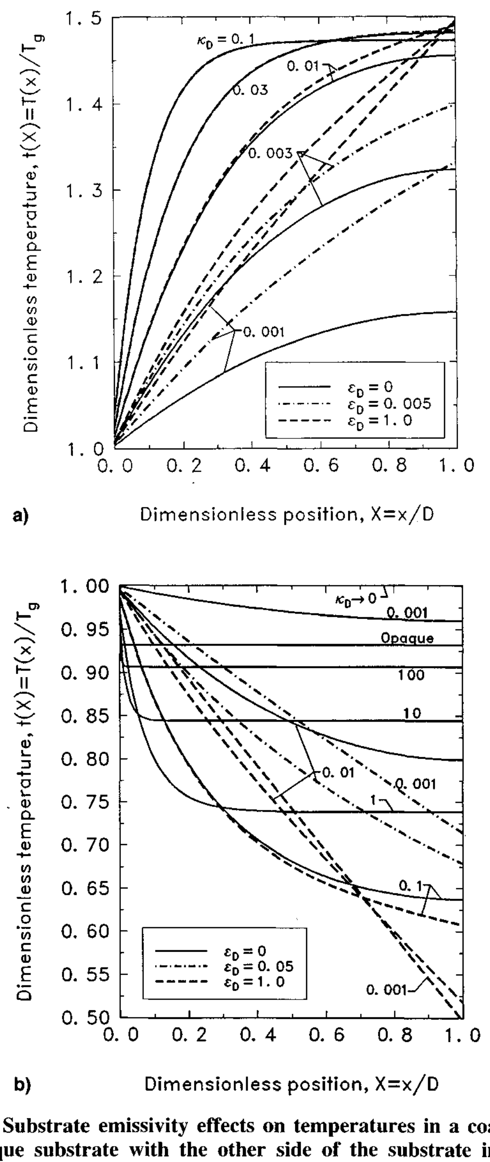

Fig. 3 Substrate emissivity effects on temperatures in a coating on an opaque substrate with the other side of the substrate insulated (n = 1.5, t_g = 1, H = 10, N = 0.1, and Ω = 0): a) radiative heating with convective cooling, $\bar{q}_r^o$ = 1.5^4 and b) convective heating with radiative cooling, $\bar{q}_r^o$ = 0.25^4.

does not depend on ε_D; Fig. 3 will show the effect of ε_D, which is found to be important only for a coating with a small optical thickness.

In Fig. 2 the largest effect of radiant absorption in the coating is for $\kappa_D \approx 0.1$, where the substrate temperature $t(1)$ is only a little below $t = 1.5$, which is the effective blackbody temperature of the surroundings. For larger or smaller κ_D the substrate temperature decreases. The limit is shown when the coating is opaque so that radiative exchange is only at $X = 0$. For very small κ_D, the low substrate temperature for $\kappa_D = 0.001$ is not realistic for actual coating behavior as it requires the substrate to be perfectly reflecting, $\varepsilon_D \to 0$; even a small ε_D provides absorption at the substrate surface and can raise its temperature considerably as shown in Fig. 3. For a large κ_D and with large convective cooling there are large temperature changes near $X = 0$ that can produce thermal stresses; the temperature gradients are reduced if N is larger as will be discussed later for some further results.

Effect of Substrate Emissivity

The previous results apply for a symmetric single layer and for a coating with the limiting condition $\varepsilon_D \to 0$. It is now shown (for the present parameters) that these results also apply for coatings with $\kappa_D > 0.03$, because for these κ_D the temperature distributions are independent of ε_D.

Figure 3a shows the effect of having a coating on a substrate with $\varepsilon_D = 1$ or 0.005; the results are compared with $t(X)$ from Fig. 2 for $\varepsilon_D = 0$. For $\varepsilon_D = 1$ and when κ_D is very small, radiation penetrates through the coating and is absorbed by the substrate. The substrate is almost at the temperature of the radiating surroundings, $t = 1.5$, and the internal temperature distribution is close to linear as in Eq. (24) as provided by heat conduction. For very small κ_D there is a substantial effect of ε_D that affects energy absorption at the substrate surface, and the results are sensitive to small ε_D as shown by $t(X)$ for $\varepsilon_D = 0.005$. The effect of ε_D is substantially decreased when $\kappa_D = 0.01$, and a further increase to $\kappa_D = 0.03$ eliminates the effect of ε_D. Hence, for the parameters considered here, when $\kappa_D > 0.03$ there is a negligible effect of ε_D relative to absorption within the coating, and the $t(X)$ from Fig. 2 apply for any ε_D. For small κ_D, such as $\kappa_D = 0.01$, there can be an increased effect of ε_D if N is larger than the $N = 0.1$ used here; this will be illustrated for $\kappa_D = 0.01$ in a later figure.

Figure 3b shows results for a coating on a substrate when the effective blackbody temperature of the radiating surroundings is small so that $q_r^o = \sigma(0.25T_g)^4$; there is then radiative cooling acting along with convective heating. For small absorption in the coating (small κ_D) and with $\varepsilon_D \to 0$, the temperature in the coating is close to T_g ($t \approx 1.0$), since there is little radiative heat loss from the coating and none from the substrate surface for these conditions. If, however, $\varepsilon_D = 1.0$, the temperature at $X = 1$ is substantially reduced for small κ_D by energy radiated from the substrate surface to the surroundings. Heat conduction provides a linear $t(X)$ as in Eq. (24) since radiation from within the coating is small. Since the convective heating is at the coating surface the highest temperature for each κ_D is at $X = 0$. As κ_D increases, the coating radiates away energy that has been conducted into its interior and $t(X)$ is no longer linear. For $\varepsilon_D \to 0$ the lowest temperature at the substrate interface $X = 1$ is for $\kappa_D \approx 0.1$; the coating is providing a maximum radiating effect. For $\varepsilon_D = 1$ the substrate temperature is further reduced by radiation loss from the substrate boundary. A small ε_D has a significant effect if $\kappa_D < 0.1$; in addition the effect of ε_D depends on the N parameter as will be discussed. For $\kappa_D > \sim 0.1$ the effect of ε_D becomes insignificant, and for larger κ_D the $t(X)$ increases toward the opaque limit. The results in Fig. 3b demonstrate that for certain ranges of κ_D and ε_D, radiation from the coating and substrate can be effective in reducing the substrate temperature.

Refractive Index Effects for Radiative Heating with Convective Cooling

The previous results are for a semitransparent material with a refractive index $n = 1.5$. Results for $n = 1$, 1.5, and 2 in Fig. 4 illustrate how $t(X)$ is influenced by n for κ_D up to the opaque limit. For $\kappa_D = 0.001$ and 0.01, $t(X)$ results are given for $\varepsilon_D = 0$ in Fig. 4a and for $\varepsilon_D = 1$ in Fig. 4b. For larger κ_D the $t(X)$ are independent of ε_D for the parameters used here and the results are the same on both parts of the figure. For small κ_D the temperatures are increased for larger n, but for $\varepsilon_D = 1$ in Fig. 4b the increase is small and the $t(1)$ are about the same. For $\kappa_D \geq 1$ the trend is reversed for most of the layer, and the maximum temperature is reduced with increasing n. This results from increased reflection at the external surface that prevents part of the incident radiation from entering the layer. For small κ_D the effect of reflecting away part of q_r^o is overcompensated by increased internal reflections providing longer radiation paths that augment internal radiative absorption and raise the internal temperatures. For $\kappa_D = 0.01$ the effect of ε_D is rather small for $n = 1.5$, as noted previously; by comparing Figs. 4a and 4b, the effect of ε_D is even smaller for $n = 2$, and it occurs in the region near the substrate. For $n = 1$ internal reflections do not occur at $X = 0$ and there is a more significant effect of ε_D.

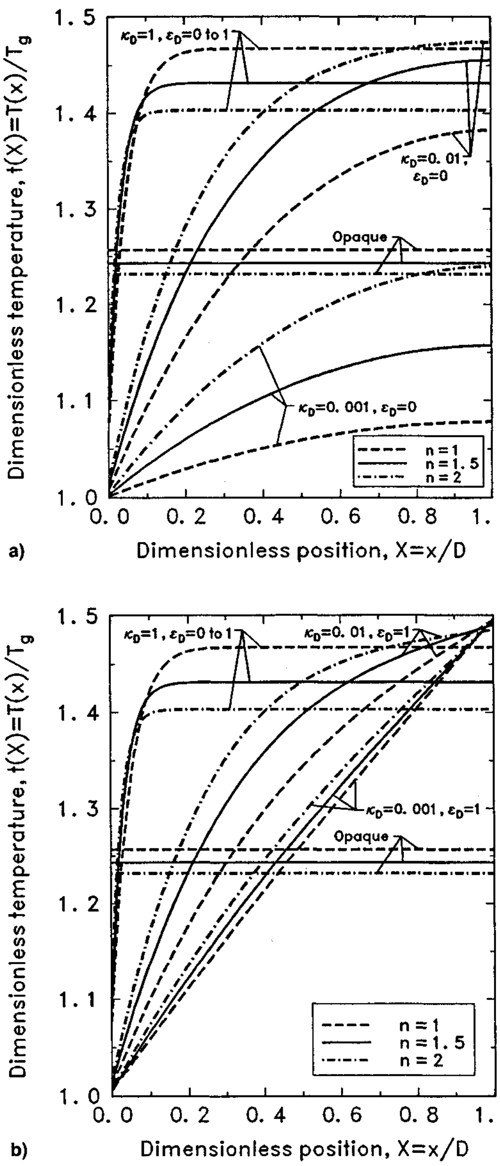

Fig. 4 Effect of coating refractive index and substrate emissivity for radiative heating with convective cooling ($t_g = 1$, $\tilde{q}_r^o = 1.5^4$, $H = 10$, $N = 0.1$, and $\Omega = 0$). Substrate emissivity, ε_D = a) 0 and b) 1.

Scattering Effects for Convective Heating with Radiative Cooling

The temperature distributions in Fig. 5 illustrate the effect of scattering; for the previous figures $\Omega = 0$. The results are for a coating with convective heating and radiative cooling. Two sets of distributions are given where aD is held constant at either 0.1 or 1 while scattering is added, thereby increasing Ω. The $t(X)$ are almost independent of ε_D for the κ_D values shown where all of the absorption optical thicknesses are $aD \geq 0.1$. For $aD = 0.1$ there is a small effect of ε_D as was shown in Fig. 3b for $\Omega = 0$; the effect decreases somewhat with increased scattering. For the heating conditions considered, increased scattering reduced the ability of the layer to radiate away energy from the coating region near the substrate. This produced higher internal temperatures and an increased substrate temperature.

Conduction and Convection Effects

The influence of the conduction and convection parameters is illustrated in Figs. 6 and 7. Figure 6 shows the very significant effects of N coupled with κ_D; Figs. 6a and 6b are, respectively, for $\kappa_D = 0.01$ and 1. The upper sets of curves where $t(X) > 1$ in Fig. 6 are for radiative heating with convective cooling. For a very small N, such as $N = 0.01$, the energy absorbed from incident radiation is not readily conducted to the external surface to be removed by convection. This causes the substrate and most of the coating to be near the surroundings temperature $t = 1.5$, and convective cooling effects are only in the region close to $X = 0$. Increasing N to 10 makes $t(X)$ rather uniform in the coating, thereby decreasing the average temperature in the coating; the substrate temperature is decreased, and near $X = 0$ the temperatures are increased. For a small $\kappa_D = 0.01$ in Fig. 6a, incident radiation can readily reach the substrate. For $\varepsilon_D = 1$ energy is absorbed at the substrate and for $N \geq 1$ there is a significant effect of ε_D. Two sets of $t(X)$ result as shown by the dot–dash and dashed curves in the upper part of Fig. 6a. For $\kappa_D = 1$ in Fig. 6b, where less incident radiation reaches the substrate, this effect becomes very small and the temperature profiles all have dt/dX close to zero at $X = 1$; hence, the results in Fig. 6b also apply for the single symmetric layer in Fig. 1a.

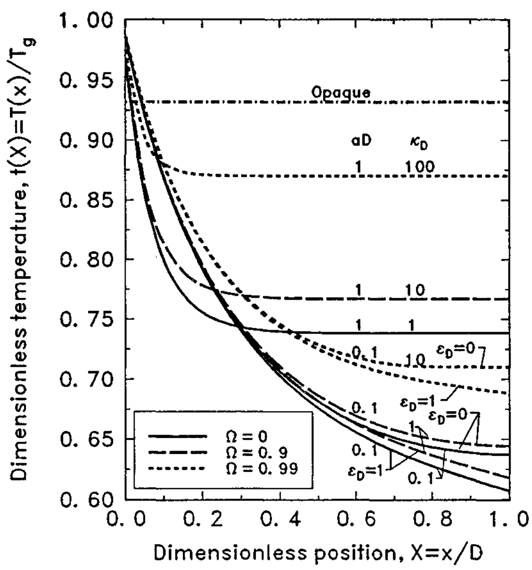

Fig. 5 Scattering effects in a coating for convective heating with radiative cooling ($n = 1.5$, $t_g = 1$, $\tilde{q}_r^o = 0.25^4$, $H = 10$, and $N = 0.1$).

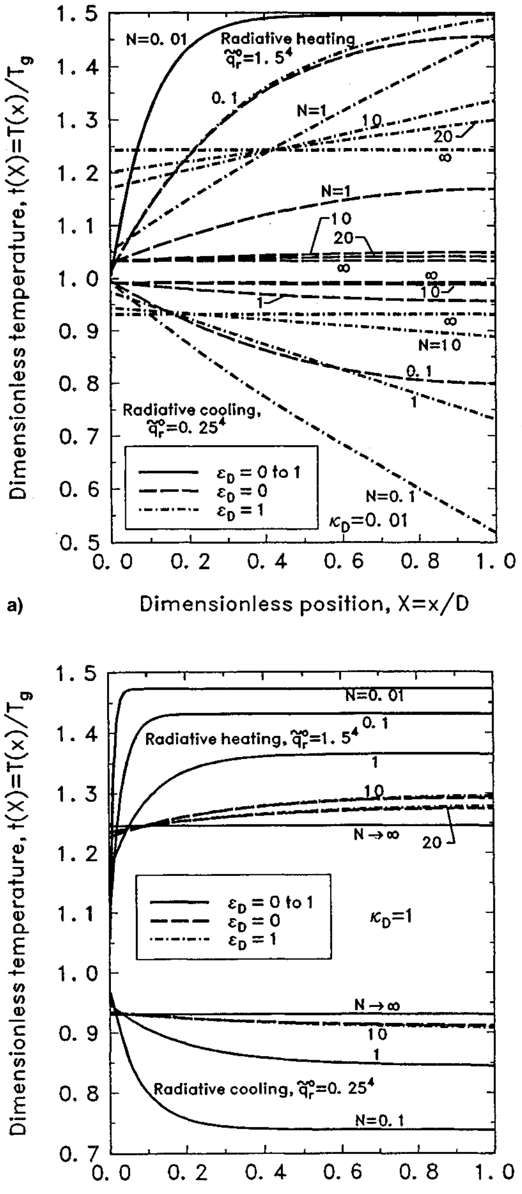

Fig. 6 Effect of heat conductivity in a coating with radiative heating and convective cooling, and for convective heating with radiative cooling (n = 1.5, t_g = 1, H = 10, and Ω = 0). Optical thickness, κ_D = a) 0.01 and b) 1.

The curves for $t(X) < 1$ in the lower parts of Figs. 6a and 6b show how radiative cooling can reduce the substrate temperature if N is small. This is especially true for small κ_D as in Fig. 6a when $\varepsilon_D = 1$. The results for $\varepsilon_D = 0$ and 1 produce two sets of curves (dashed and dot–dashed) in the lower part of Fig. 6a for $\kappa_D = 0.01$. However, in Fig. 6b for $\kappa_D = 1$ there is very little effect of ε_D. For $N = 10$ the $t(X)$ results approach the uniform temperature limits from Eq. (23).

An increase in the external convection coefficient (the H parameter) produces increased temperature gradients near the outer boundary; these can become large as in Fig. 7 where $N = 0.1$. As shown by Fig. 6, other N values will substantially change the $t(X)$ results. For $\kappa_D = 1$ the temperatures are uniform over much of the coating interior, and the results are valid for any ε_D. For $\kappa_D = 0.003$ two sets of curves show the effect of ε_D, and there is a substantial difference between a black and perfectly reflecting substrate. The $t(X)$ in Fig. 7 for $\kappa_D = 0.003$ with $\varepsilon_D = 0$, and for $\kappa_D = 1$ for any ε_D, are also valid for a symmetric single layer (Fig. 1a).

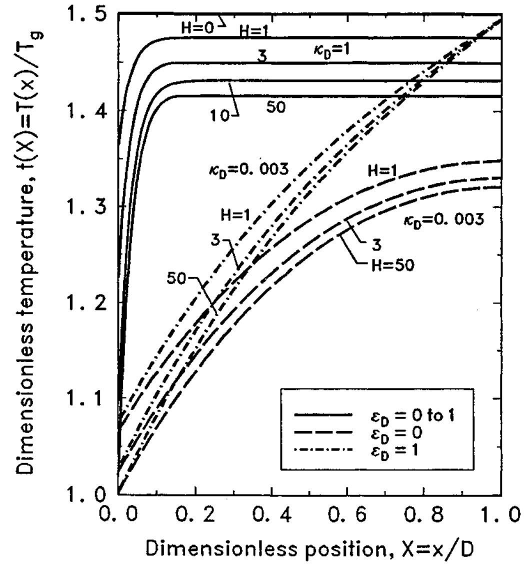

Fig. 7 External convective cooling effects on temperatures in a coating heated by radiation (κ_D = 0.003 and 1, n = 1.5, t_g = 1, $\tilde{q}_r^o$ = 1.5^4, N = 0.1, and Ω = 0).

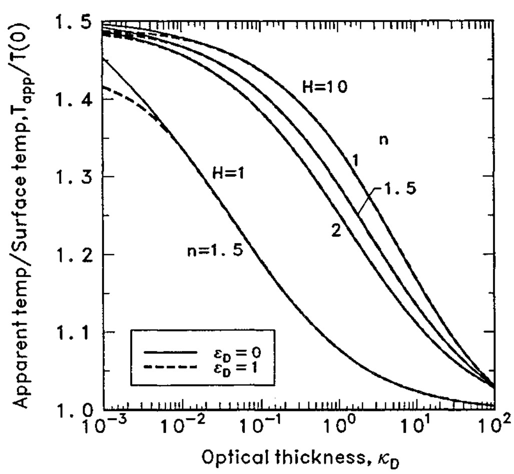

Fig. 8 Apparent surface temperature determined from radiation leaving the coating surface (t_g = 1, $\tilde{q}_r^o$ = 1.5^4, H = 1 and 10, N = 0.1, and Ω = 0).

Apparent Surface Temperature

For measuring the surface temperature of a coating with a radiation detector, it is of interest to compare the temperature calculated from the radiative flux leaving from within the coating with the actual surface temperature. The results for $H = 10$ in Fig. 8 are for large convection, and since conduction is small ($N = 0.1$), there are large variations in $t(X)$ within the coating. Since the interior temperatures are larger than $t(0)$, the T_{app} calculated from the exiting radiation is larger than $T(0)$. The optical thickness must be quite large for $T_{app}/T(0)$ to be near 1. For $H = 1$ the gradients of $t(X)$ are reduced and T_{app} is closer to $T(0)$; this would also be true for a larger N as in Fig. 6. The effects of H, N, and ε_D are readily examined using the present analytical relations. For radiative cooling with convective heating the $T_{app}/T(0) < 1$.

Conclusions

A convenient analytical expression has been derived to predict thermal behavior of a semitransparent coating on an opaque substrate when internal cooling is not provided. The substrate is either insulated on its side away from the coating, or the geometry and boundary conditions are symmetric, and so there is no net heat flow through the coated material. The temperature distribution in the coating is a result of the local balance between internal radiation and conduction. When the coating is heated by hot gases in cooled surroundings, there can be a reduction in substrate temperature as a result of radiative cooling from within the coating. For most of the illustrative results given here, a coating optical thickness larger than about $\kappa_D = 0.1$ gives a thermal behavior almost independent of the substrate emissivity. This value of κ_D is, however, sensitive to the parameters used such as the amount of heat conduction. When the results are independent of ε_D they also apply for a single semitransparent layer with symmetric boundary conditions. For very optically thin coatings the temperature distributions can be quite sensitive to the substrate emissivity. Results for other parameters are easily examined by using the convenient analytical expressions that are provided for arbitrary parameters and some limiting cases.

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